
Student Number

2023 Glenwood High School
Year 12 - Trial HSC Examination

Mathematics Advanced

General Instructions	• Reading Time – 10 minutes • Working time – 3 hours • Write using black pen • NESA approved calculators may be used • A reference sheet will be provided • For questions in Section II, show relevant mathematical reasoning and/or calculations
Total marks: 100	**Section I – 10 marks** (pages 2 – 7) • Attempt Questions 1- 10 • Allow about 15 minutes for this section **Section II – 90 marks** (pages 9 – 32) • Attempt Questions 11 – 33 • Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

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1. Given $x + \frac{1}{x} = 4$, what is $x^2 + \frac{1}{x^2}$?

A. 14

B. 16

C. 18

D. 20

2. The number of hours worked during a week by casual staff in a reception centre is normally distributed with a mean of 16 hours and a standard deviation of 2.5 hours.

What is the percentage of casual staff working fewer than 21 hours in a week?

A. 47.5%

B. 84%

C. 95%

D. 97.5%

3. pH measures the concentration of hydrogen ions, $[\mathrm{H}^+]$, in a liquid solution. The formula to calculate the pH of a solution is given by

$$\mathrm{pH} = -\log_{10}[\mathrm{H}^+]$$

What is the concentration of hydrogen ions in a solution with a pH of 1.5?

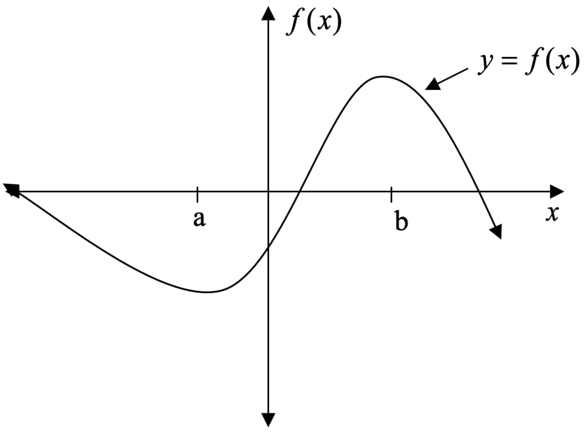
A. 1.5^{-10}

B. 1.5^{10}

C. $10^{-1.5}$

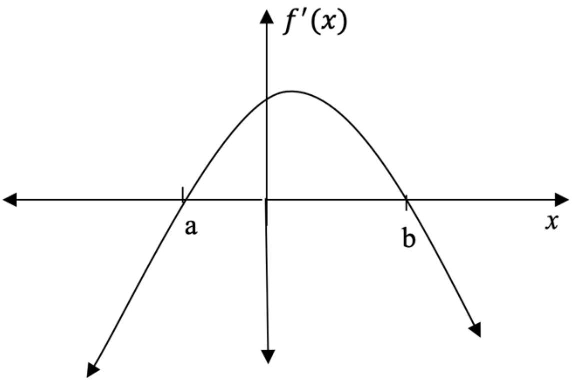
D. $10^{1.5}$

4. The diagram shows the graph $y = f(x)$.

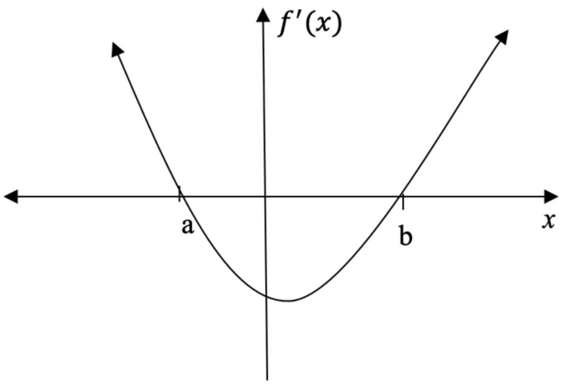


Which of the following graphs shows $y = f'(x)$.

A.



B.



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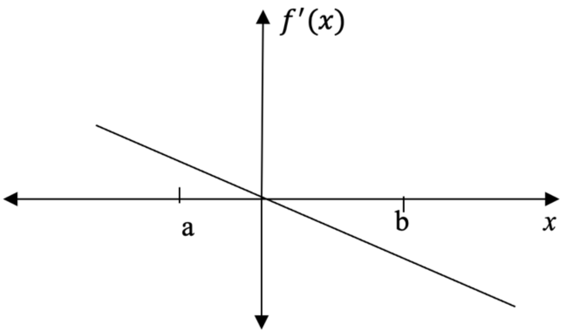
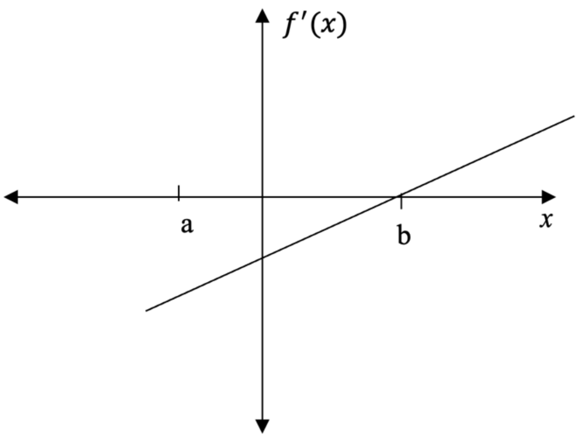
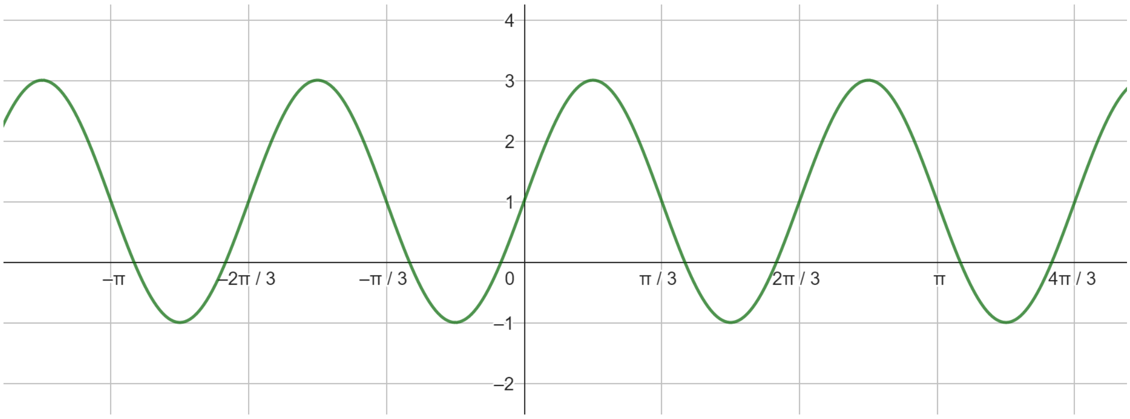


D.



5. In the diagram, a graph of a trigonometric function is given.



Which of the following could be the equation of the given graph?

A. $y = 2\sin\left(3x - \frac{\pi}{2}\right) + 1$

B. $y = 2\sin(2x - \pi) + 1$

C. $y = 2\cos\left(3x - \frac{\pi}{2}\right) + 1$

D. $y = 2\cos\left(2x + \frac{\pi}{2}\right) + 1$

6. What is $\int \frac{4}{(2x-1)^2}\,dx$?

A. $2\ln(2x - 1) + c$

B. $\frac{-2}{2x - 1} + c$

C. $\frac{-2}{(2x - 1)^3} + c$

D. $\frac{8}{2x - 1} + c$

7. The curve $y = \ln x$ is translated to the left by π units and then dilated horizontally by a scale factor of 3.

What is the equation which describes this new curve?

A. $y = \ln\left(\dfrac{x+\pi}{3}\right)$

B. $y = \ln\left(\dfrac{x}{3} - \pi\right)$

C. $y = 3\ln(x + \pi)$

D. $y = \ln\left(\dfrac{x}{3} + \pi\right)$

8. Differentiate $x\sin\left(\dfrac{1}{x}\right)$

A. $\sin\left(\dfrac{1}{x}\right) - \dfrac{1}{x}\cos\left(\dfrac{1}{x}\right)$

B. $\cos\left(\dfrac{1}{x}\right) - \dfrac{1}{x}\sin\left(\dfrac{1}{x}\right)$

C. $\sin\left(\dfrac{1}{x}\right) + \dfrac{1}{x}\cos\left(\dfrac{1}{x}\right)$

D. $x\cos\left(\dfrac{1}{x}\right)$

9. The domain of the function $f(x)$ is $[-2,\infty)$.

What is the domain of the function $3f(-2x) - 4$?

A. $[1,\infty)$

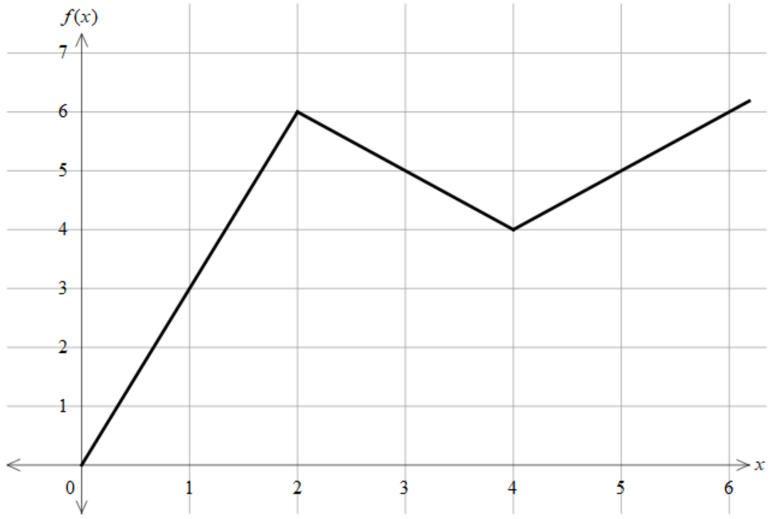
B. $(-\infty,-4]$

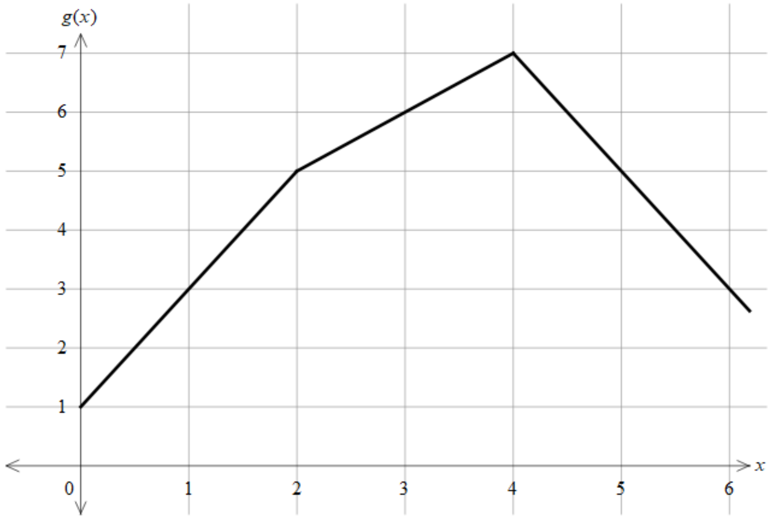
C. $(-\infty,1]$

D. $[-4,\infty)$

10. The graphs of the functions $f(x)$ and $g(x)$ are displayed below.

Let $I(x) = f(g(x))$ and $J(x) = g(f(x))$. Which of the following has the highest value?





A. $I'(1)$

B. $J'(1)$

C. $I'(5)$

D. $J'(5)$

END OF SECTION I

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2023 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Student Number

Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11 – 33
Allow about 2 hour and 45 minutes for this section

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (4 marks)

A group of scientists went to a forest near a riverbank to determine if there is a correlation between the number of plant species and the distance from the river.

The results are summarised in the following table.

Distance from the riverbank (m)	4	9	11	15	17	22	28
Number of plant species	26	22	18	14	12	11	9

(a) Calculate Pearson's correlation coefficient for the data giving your answer to four decimal places. **1**

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(b) Find the equation of the least-squares regression line in the form of $y = A + Bx$. **2**
Give the values of A and B to three decimal places.

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(c) At a certain distance from the riverbank, Max counted 16 plant species. **1**

Calculate the predicted distance, to the nearest metre, from the riverbank using the equation of the least-squares regression line.

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Question 12 (2 marks)

Differentiate $y = \ln(x+3)^2$ **2**

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Question 13 (3 marks)

Describe three transformations which, when applied in succession, change the graph of $y = x^2 + 5$ to the graph with equation $y = \left(\frac{x+1}{2}\right)^2$. **3**

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Question 14 (4 marks)

(a) Differentiate $\dfrac{x^2}{x^2+1}$ **2**

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(b) Hence, evaluate $\displaystyle\int_1^2 \frac{x}{(x^2+1)^2}\,dx$ **2**

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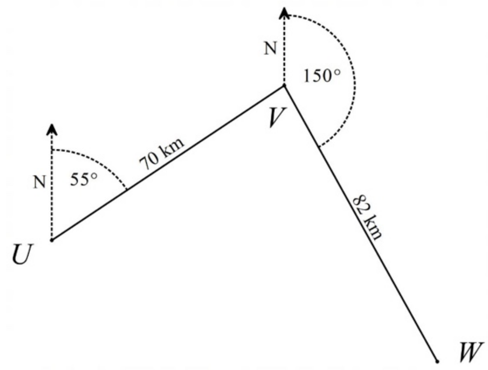
Question 15 (4 marks)

A helicopter leaves Underwood and flies 70 km on a bearing of 055° to Vanna Beach.

It then flies 82 km on a bearing of 150° to Weston.

The diagram to the right illustrates the journey.



(a) The helicopter then plans to fly directly back to Underwood. **2**

Calculate, to the nearest km, the distance that it will fly.

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(b) Determine the bearing (from Weston) on which it should fly to return to Underwood. **2**

Give your answer to the nearest degree.

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Question 16 (5 marks)

A survey found that in a large population approximately 25% of people are left-handed.

(a) Three people are selected at random. Find the probability that at least one of them is left-handed. **2**

(b) What is the smallest number of people that would need to be selected to have a greater than 99% chance that at least one of them is left-handed. **3**

Question 17 (3 marks)

The table below shows the probability distribution of a discrete random variable X.

x	0	2	4	5	8	9
$P(X=x)$	k^2	0.16	0.18	0.3	k	0.12

(a) Find the value of k. **2**

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(b) Calculate $E(X)$. **1**

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Question 18 (2 marks)

A sector has radius length of 20 cm and the angle subtended at the centre is $50°$. The radius of this sector is increased by 25% and its angle at the centre is decreased by $k\%$. If the area of the sector remains unchanged, find the value of k. **2**

Question 19 (4 marks)

The table below gives the future value of an annuity of $1 per period for various periods and interest rates.

Table of Future Value Interest Factors								
	Interest rate per period							
Number of Periods	0.25%	0.30%	0.35%	0.40%	0.45%	0.50%	0.55%	0.60%
53	56.5961	57.3530	58.1230	58.9063	59.7033	60.5141	61.3391	62.1785
54	57.7376	58.5250	59.3264	60.1419	60.9719	61.8167	62.6765	63.5516
55	58.8819	59.7006	60.5340	61.3825	62.2463	63.1258	64.0212	64.9329
56	60.0291	60.8797	61.7459	62.6280	63.5264	64.4414	65.3733	66.3225
57	61.1792	62.0624	62.9620	63.8786	64.8123	65.7636	66.7329	67.7204
58	62.3322	63.2485	64.1824	65.1341	66.1040	67.0924	68.0999	69.1267
59	63.4880	64.4383	65.4070	66.3946	67.4014	68.4279	69.4744	70.5415
60	64.6467	65.6316	66.6359	67.6602	68.7047	69.7700	70.8565	71.9647
61	65.8083	66.8285	67.8692	68.9308	70.0139	71.1189	72.2463	73.3965
62	66.9729	68.0290	69.1067	70.2065	71.3290	72.4745	73.6436	74.8369
63	68.1403	69.2331	70.3486	71.4874	72.6499	73.8368	75.0487	76.2859
64	69.3106	70.4408	71.5948	72.7733	73.9769	75.2060	76.4614	77.7436
65	70.4839	71.6521	72.8454	74.0644	75.3098	76.5821	77.8820	79.2101
66	71.6601	72.8670	74.1004	75.3607	76.6487	77.9650	79.3103	80.6854

(a) Julia invests $250 per month in an annuity which pays 5.4% p.a. compounding monthly. What will be the value of the annuity after 5 years? **2**

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(b) Carrington finds that he can only get an interest rate of 3.6% p.a. compounding monthly. **2**

If he wants to achieve the same total amount as Julia after the same period, what amount should he invest each month?

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Question 20 (5 marks)

A random variable is normally distributed with mean 0 and standard deviation 1.

The table gives the probability that this random variable is less than z for different values of z.

z	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2
$P(Z < z)$	0.0228	0.0668	0.1587	0.3085	0.5000	0.6915	0.8413	0.9332	0.9772

(a) Using the table, find the probability that a value from a random variable that is normally distributed with mean 0 and standard deviation 1, lies between -0.5 and 1. **1**

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(b) The blood pressure readings of patients with a certain condition are normally distributed, with a mean of μ and a standard deviation of σ. It is known that 2.28% of these patients have a blood reading of less than 105 and 6.68% of these patients have a blood reading of more than 133. Using the table, find μ and σ . **4**

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Question 21 (3 marks)

Without using calculus, sketch the graph of $y = 2 - \frac{1}{x-4}$, clearly labelling asymptotes and x and y intercepts. **3**

Question 22 (3 marks)

Solve $\log_2 x + \log_2 (x-3) = 2$ **3**

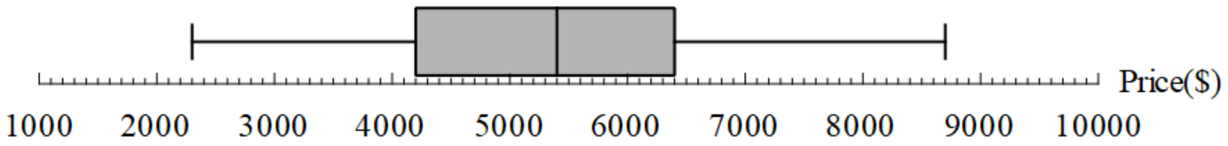
Question 23 (3 marks)

Evaluate **3**

$$\int_2^3 \frac{2x\,dx}{3x^2 - 4}$$

Question 24 (5 marks)

There are 30 paintings in a warehouse. The box-and-whisker diagram below shows the prices of the paintings inside the warehouse.



(a) Find the interquartile range. **1**

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(b) Show that there are no outliers. **2**

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(c) It is given that the mean is \$5500. Four paintings of respective prices \$3400, \$3500, \$5700 and \$6800 are now donated to an art gallery. Find the mean and median of the prices of the remaining paintings in the warehouse. **2**

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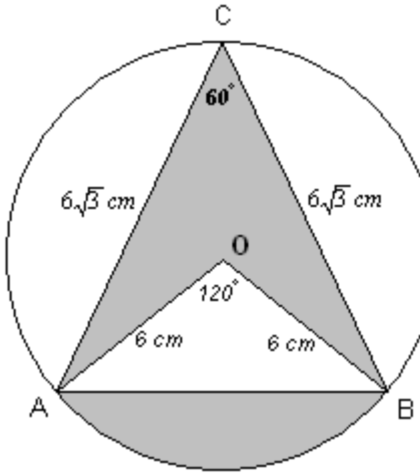
Question 25 (3 marks)

The design drawing for a company logo is shown. The circle has centre O and radius 6 cm as shown in the diagram.

$AC = CB = 6\sqrt{3}$ cm,

$\angle AOB = 120°$ and $\angle ACB = 60°$



Show that the exact area of the shaded region is $12\pi + 9\sqrt{3}$ cm^2. **3**

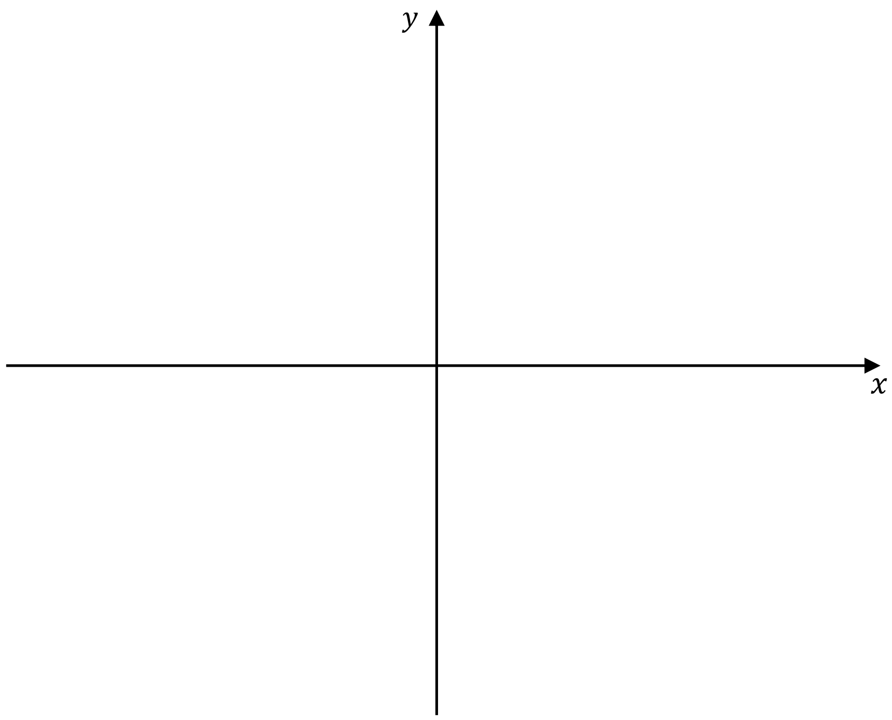
Official Use Only – Do NOT write anything, or make a mark below this line

Question 26 (4 marks)

Find the global maximum and minimum values of $f(x) = \sin^2 x + 4\cos x$, where $\frac{\pi}{3} \leq x \leq \frac{3\pi}{2}$. **4**

Question 27 (3 marks)

By factorising or otherwise, sketch the graph of $y = x^3 - 5x^2 - 4x + 20$ showing all intercepts with the axes. **You do not need to find any stationary points.** 3



Question 28 (4 marks)

(a) If x is an acute angle, prove that $(1-\sin x)(\sec x+\tan x)\equiv\cos x$. **2**

(b) Hence or otherwise, evaluate **2**

$$\int_0^{\frac{\pi}{4}} \sin^2 x(1-\sin x)(\sec x+\tan x)\,dx$$

Question 29 (6 marks)

The continuous random variable t, represents the time it takes, in years, to construct a high-rise building. The probability density function for t is given by

$$f(t) = \begin{cases} kt(5-t) & 0 \le t \le 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k = \frac{6}{125}$ **2**

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(b) Find the mode of the distribution. **2**

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(c) What is the percentage of high-rise buildings constructed within a year? **2**

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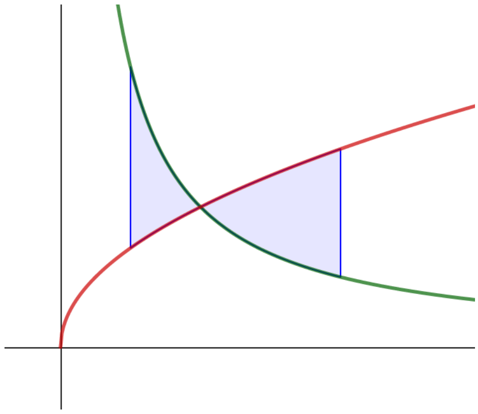
Question 30 (4 marks)

In the diagram, the region between the graphs of $y = \frac{4}{x}$ and $y = \sqrt{2x}$ for $1 \leq x \leq 4$ is shaded. **4**



Find the area of the shaded region.

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Question 31 (6 marks)

Find and classify the nature of any stationary points and points of inflections for the function $y = x^2 e^x$. **6**
Hence, sketch the graph of the function, clearly labelling any stationary points, points of inflection and intercepts.

Question 32 (3 marks)

Two tangents can be drawn from the origin to the graph of the curve $y = x^2 + 3x + 5$. **3**
Find the equations of these tangents.

Question 33 (7 marks)

The population size P of a species of birds living in a wildlife preserve increases at a rate of

$$\frac{dP}{dt} = 9e^{\frac{t^2}{5}} - 2t \quad \text{for } t \geq 0,$$

where t is the time in months. It is known that the initial population of the birds is 34.

(a) Use the trapezoidal rule with 4 sub intervals to estimate $\int_0^4 e^{\frac{t^2}{5}}\, dt$. **3**

Hence, show that $P = 218$ (to the nearest integer) when $t = 4$.

Question 33 continues next page

A coal mine was recently built near the wildlife preserve and pollution from the mine affects the population of the birds from $t = 4$ onwards. An environmentalist models that the population size of the birds can then be approximated by $P = Ate^{-0.05t} - 100, \quad (t \geq 4)$.

(b) Using part (a), show that $A = 97$. **1**

(c) Determine the maximum population size, leaving your answer as an integer. **3**

END OF EXAM

Section II Extra writing space

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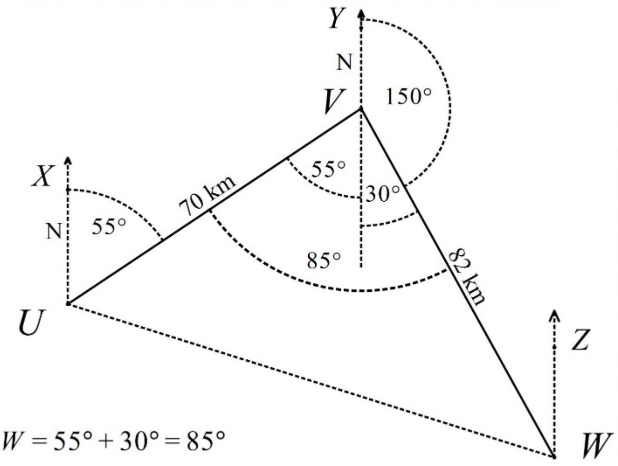
Yr 12 Adv Trial 2023 Solutions

Multiple choice	
Q1	A
Q2	D
Q3	C
Q4	A
Q5	C
Q6	B
Q7	D
Q8	A
Q9	C
Q10	B

Q11a	$r = -0.9475$
Q11b	$A = 27.045$ $B = -0.729$ $y = A + Bx$ $y = 27.045 - 0.729x$
Q11c	$16 = 27.045 - 0.729x$ $x = 15\text{ m}$
Q12	$y = \ln(x+3)^2$ $y = 2\ln(x+3)$ $\frac{dy}{dx} = \frac{2}{(x+3)}$ OR $y = \ln(x+3)^2$ $\frac{dy}{dx} = \frac{2(x+3)}{(x+3)^2}$ $\frac{dy}{dx} = \frac{2}{(x+3)}$
Q13	$x^2 + 5 \to x^2 \to \left(\frac{x}{2}\right)^2 \to \left(\frac{x+1}{2}\right)^2$ Translation 5 units down Horizontal dilation scale factor 2 Translation 1 unit left
Q14a	$\frac{2x(x^2+1) - x^2 \times 2x}{(x^2+1)^2} = \frac{2x}{(x^2+1)^2}$
Q14b	$\int_1^2 \frac{x}{(x^2+1)^2}\,dx = \frac{1}{2}\left[\frac{x^2}{x^2+1}\right]_1^2 = \frac{1}{2}\left(\frac{4}{5} - \frac{1}{2}\right) = \frac{3}{20}$

Q15a

Let X, Y and Z be points due north of U, V and W respectively



$\angle UVW = 55° + 30° = 85°$

Using the cosine rule:

$UW^2 = 70^2 + 82^2 - 2 \times 70 \times 82 \times \cos(85°)$

$= 10\,623.452$

$UW = 103.07$

$UW = 103$ km (nearest km)

Q15b

The sine rule can be used to find the angle inside the triangle *UVW*.

Two cases are shown below.

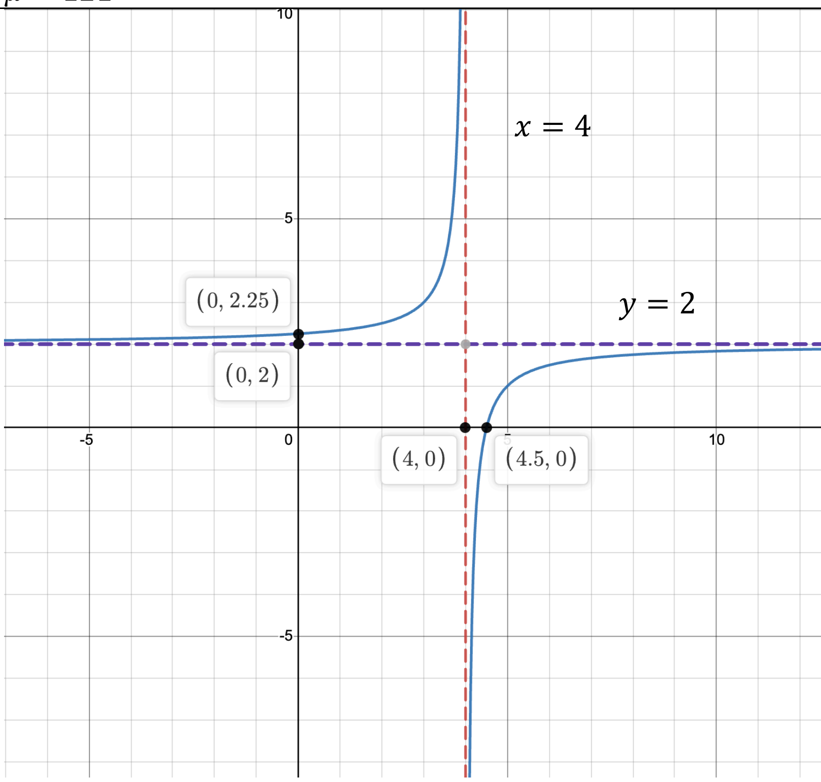
$$\frac{\sin\ \angle VWU}{70} = \frac{\sin 85°}{103.07}$$
$$\sin\ \angle VWU = \frac{70 \times \sin 85°}{103.07}$$
$$= 0.67656$$
$$\angle VWU = \sin^{-1}(0.67656)$$
$$= 42.575$$
$$\angle ZWV = 30° \quad \text{(alternate angles)}$$
$$\angle UWZ = 30 + 43 = 73°$$
$$\text{Bearing required} = 360 - 73 = 287°$$

$$\frac{\sin\ \angle VUW}{82} = \frac{\sin 85°}{103.7}$$
$$\sin\ \angle VUW = \frac{82 \times \sin 85°}{103.7}$$
$$= 0.7925$$
$$\angle VUW = \sin^{-1}(0.7925)$$
$$= 52.4242$$
$$\angle XUW = 55 + 52 = 107$$
$$\angle UWZ = 180 - 107 = 73°$$
$$\text{Bearing required} = 360 - 73 = 287°$$

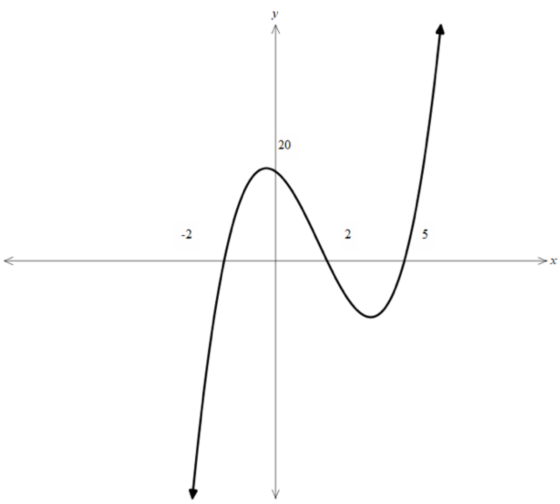
The angles can also be found using the Cosine rule, one example is shown below.

$$\cos\ (\angle VWU) = \frac{82^2 + 103.07^2 - 70^2}{2 \times 82 \times 103.07}$$
$$= 0.73638238$$
$$\angle VWU = \cos^{-1}(0.73638238)$$
$$= 42.57584516$$
$$\angle ZWV = 30° \quad \text{(alternate angles)}$$
$$\angle UWZ = 30 + 43 = 73°$$
$$\text{Bearing required} = 360 - 73 = 287°$$

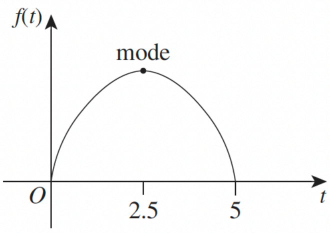
Q16a	Let L stand for left-handed and N for not left-handed. P(L)=25% P(N)=75% P(at least one left-handed)=1-P(NNN) $= 1 - (0.75)^3 = \frac{37}{64}$
Q16b	$1-(0.75)^n>0.99$ $1-0.99>(0.75)^n$ $0.01>(0.75)^n$ $\ln(0.01)>n\ln(0.75)$ $n>\dfrac{\ln(0.01)}{\ln(0.75)}$ $n>16.0078$ ∴17 people
Q17a	$k^2+0.16+0.18+0.3+k+0.12=1$ $k^2+k+0.76=1$ $k^2+k-0.24=0$ $(k-0.2)(k+1.2)=0$ $\therefore k=0.2$ as $k>0$
Q17b	$E(X)=0\times0.2^2+2\times0.16+4\times0.18+5\times0.3+8\times0.2+9\times0.12$ $E(X)=5.22$
Q18	$\frac{1}{2}\times 20^2\times\frac{50\pi}{180}=\frac{1}{2}\times 25^2\times\frac{\theta\pi}{180}$ $20000 = 625\theta$ $\theta = 32$ % difference $= \frac{50-32}{50}\times 100 = 36\%$ $k = 36$
Q19a	5.4% pa compounding monthly is 5.4% ÷ 12 =0.45% per month 5 years is 5×12 = 60 months From the table the $1 factor for 0.45% for 60 months is 68.7047 For $250, the annuity is worth 250 × 68.7047 = $17 176.18
Q19b	If annual rate is 3.6% this is 3.6 ÷ 12=0.30% per month From the table the $1 factor for 0.30% for 60 months is 65.6316 If investment is to earn $17176 with a factor of 65.6316 then monthly annuity is 17176 ÷ 65.6316 = $261.70320394 =$261.71 per month

Q20a	$0.8413 - 0.3085 = 0.5328$
Q20b	$2.28\% = P(Z < -2)$ $\Rightarrow \dfrac{105 - \mu}{\sigma} = -2$ $100\% - 6.68\% = 93.32\% = P(Z < 1.5)$ $\Rightarrow \dfrac{133 - \mu}{\sigma} = 1.5$ Solving these equations simultaneously $133 - \mu = 1.5\sigma - (1)$ $105 - \mu = -2\sigma - (2)$ $(1) - (2)$ $28 = 3.5\sigma$ $\sigma = 8$ $\mu = 133 - 1.5 \times 8$ $\mu = 121$
Q21	
Q22	$\log_2 x + \log_2(x - 3) = 2$ $log_2(x^2 - 3x) = log_2 4$ $x^2 - 3x - 4 = 0 \Rightarrow \begin{bmatrix} x = -1 \\ x = 4 \end{bmatrix}$ From domain, $x > 3$ Therefore, solution is $x = 4$

Q23	$\int_2^3 \frac{2xdx}{3x^2-4}$ $=\frac{1}{3}\int_2^3 \frac{6xdx}{3x^2-4}$ $=\frac{1}{3}[\log_e(3x^2-4)]_2^3$ $=\frac{1}{3}[\log_e(23)-\log_e(8)]$ $=\frac{1}{3}\log_e\left(\frac{23}{8}\right)$
Q24a	$Q_3 = 6400$ $Q_1 = 4200$ $IQR = 6400 - 4200$ $IQR = 2200$
Q24b	$\text{min} = 2300$ $\text{max} = 8700$ $Q_1 - 1.5\times IQR = 4200 - 1.5\times 2200$ $= 900 < 2300$ $Q_3 + 1.5\times IQR = 6400 + 1.5\times 2200$ $= 9700 > 8700$ $\therefore$ no outliers
Q24c	$\text{median} = 5400$ 2 paintings below and 2 paintings above the median are removed $\therefore$ median remains unchanged i.e. median = \$5400 total sum of prices $= 5500\times 30$ $= 165000$ $\therefore$ new total $= 165000 - 3400 - 3500 - 5700 - 6800$ $= 145600$ new mean $= \frac{145600}{26}$ $= \$5600$

Q25	Area of segment = area of sector AOB – area of triangleAOB $Area\, of\ segment = \frac{1}{2}(6)^2\frac{2\pi}{3} - \frac{1}{2}(6)^2 \sin\frac{2\pi}{3}$ $= 12\pi - 9\sqrt{3}$ $Area\, of\ AOBC = \frac{1}{2}\sqrt{108}\sqrt{108}\sin 60 - \frac{1}{2}(6)(6)\sin 120$ $= 27\sqrt{3} - 9\sqrt{3}$ $Total\, area = 12\pi - 9\sqrt{3} + 27\sqrt{3} - 9\sqrt{3}$ $= 12\pi + 9\sqrt{3}\, cm^2$
Q26	$f'(x) = 2\sin x\cos x - 4\sin x = 2\sin x(\cos x - 2)$ $f'(x) = 0 \Rightarrow \sin x = 0 \Rightarrow x = \pi$ $f\left(\frac{\pi}{3}\right) = \frac{11}{4}, f(\pi) = -4, f\left(\frac{3\pi}{2}\right) = 1$ $f_{\min} = -4, f_{\max} = \frac{11}{4}$
Q27	$y = x^3 - 5x^2 - 4x + 20$ $y = x^2(x-5) - 4(x-5)$ $y = (x^2 - 4)(x-5)$ $y = (x-2)(x+2)(x-5)$ 

Q28a	$LHS:(1-\sin x)\left(\frac{1}{\cos x}+\frac{\sin x}{\cos x}\right)$ $\equiv(1-\sin x)\left(\frac{1+\sin x}{\cos x}\right)$ $\equiv\left(\frac{1-\sin^2 x}{\cos x}\right)$ $\equiv\left(\frac{\cos^2 x}{\cos x}\right)$ $\equiv\cos x$ $=RHS$
Q28b	From a) $\int_0^{\frac{\pi}{4}} \sin^2 x(1-\sin x)(\sec x+\tan x)\,dx=\int_0^{\frac{\pi}{4}} \sin^2 x\cos x\,dx$ $=\left[\frac{1}{3}\sin^3 x\right]_0^{\frac{\pi}{4}}$ $=\frac{1}{3}\left[\sin^3\left(\frac{\pi}{4}\right)-\sin^3(0)\right]$ $=\frac{1}{3}\left[\left(\frac{1}{\sqrt{2}}\right)^3-0\right]$ $=\frac{1}{6\sqrt{2}}$ $=\frac{\sqrt{2}}{12}$

Q29a	Since $\int_{-\infty}^{\infty} f(t)\,dt = 1$: $\int_0^5 kt(5-t)\,dt = 1$ $\int_0^5 5t - t^2\,dt = \frac{1}{k}$ $\left[\frac{5t^2}{2} - \frac{t^3}{3}\right]_0^5 = \frac{1}{k}$ $\left(\frac{5\times 5^2}{2} - \frac{5^3}{3}\right) = \frac{1}{k}$ $\frac{125}{6} = \frac{1}{k}$ $k = \frac{6}{125}$	
Q29b	The mode is the value of t that gives the maximum value of $f(t)$. Since $f(t) = \frac{6}{125}t(5-t)$ is concave down, the maximum value occurs at the axis of symmetry. Substituting $f(t) = 0$ to find the roots of the parabola gives: $\frac{6}{125}t(5-t) = 0$ $t = 0, 5$ Given that the axis of symmetry is the midpoint between these values:  The axis of symmetry is at $t = 2.5$ and, hence, the mode is $t = 2.5$.	**Or** Find turning point $f(t) = \frac{6t}{25} - \frac{6t^2}{125}$ $f'(t) = 0$ $f'(t) = \frac{6}{25} - \frac{12t}{125}$ $\frac{6}{25} - \frac{12t}{125} = 0$ $t = 2.5$

Q29c	$P(t \le 1) = \int_0^1 \frac{6}{125} t(5-t)dt$ $= \frac{6}{125} \int_0^1 5t - t^2 dt$ $= \frac{6}{125} \left[\frac{5t^2}{2} - \frac{t^3}{3} \right]_0^1$ $= \frac{6}{125} \left[\left(\frac{5}{2} - \frac{1}{3} \right) - (0-0) \right]$ $= \frac{13}{125}$ $= 0.104$ Therefore, 10.4% of the high-rise buildings are constructed within a year.
Q30	$\frac{4}{x} = \sqrt{2x} \Rightarrow \frac{16}{x^2} = 2x \Rightarrow 16 = 2x^3 \Rightarrow x = 2$ Area $= \int_1^2 \left(\frac{4}{x} - \sqrt{2x} \right) dx + \int_2^4 \left(\sqrt{2x} - \frac{4}{x} \right) dx$ $= \left[4\ln x - \frac{2\sqrt{2}}{3}\sqrt{x^3} \right]_1^2 + \left[\frac{2\sqrt{2}}{3}\sqrt{x^3} - 4\ln x \right]_2^4$ $= 4\ln 2 - \frac{8}{3} - 0 + \frac{2\sqrt{2}}{3} + \frac{16\sqrt{2}}{3} - 4\ln 4 - \frac{8}{3} + 4\ln 2$ $= 6\sqrt{2} - \frac{16}{3}$ $= 3.152$

Q31	$y = x^2e^x$ $u = x^2 \quad v = e^x$ $u' = 2x \quad v' = e^x$ $y' = 2xe^x + x^2e^x$ or $y' = xe^x(2+x)$ $u = 2x \quad v = e^x$ $u' = 2 \quad v' = e^x$ $y'' = 2e^x + 2xe^x + 2xe^x + x^2e^x$ $y'' = 2e^x + 4xe^x + x^2e^x$ $y'' = e^x(2 + 4x + x^2)$ stat pt $y' = 0$ $0 = xe^x(2+x)$ $\therefore \; x = 0$, $e^x = 0$ (DNE) , $2 + x = 0$ $\therefore \; x = 0$ or $x = -2$ when $x = 0$ $y = 0$, (0,0) when $x = -2$ $y = \frac{4}{e^2}$ $\left(-2, \frac{4}{e^2}\right)$ classify $x = 0$ $y'' = 1(2+0) > 0$ concave up $\therefore$ min pt (0,0) $x = -2$ $y'' = e^{-2}(2-8+4) < 0$ concave down $\therefore$ max pt $\left(-2, \frac{4}{e^2}\right)$

Inflections $y'' = 0$

$$0 = e^x(2 + 4x + x^2)$$

$$\therefore \quad e^x = 0 \ (DNE)$$

or

$$x^2 + 4x + 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 2}}{2}$$

$$x = \frac{-4 \pm \sqrt{8}}{2}$$

$$x = \frac{-4 \pm 2\sqrt{2}}{2}$$

$$x = \frac{\cancel{2}\left(-2 \pm 2\sqrt{2}\right)}{\cancel{2}}$$

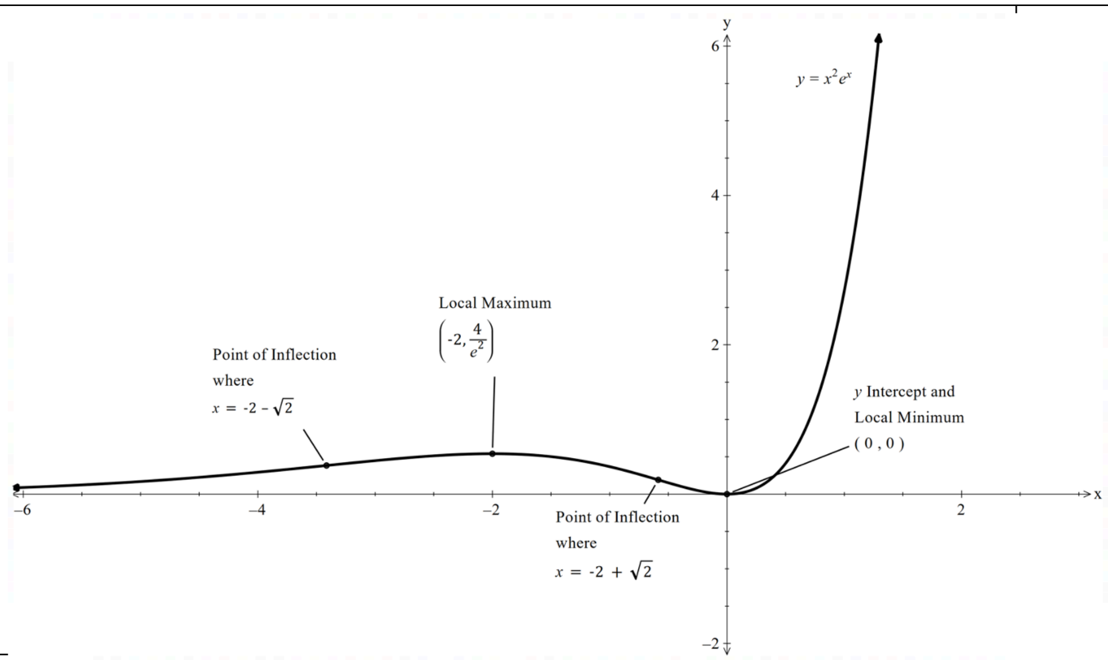
$$x = -2 + \sqrt{2} \ \text{ or } -2 - \sqrt{2}$$

-1	-0.6	0
<0	0	>0

Concavity changes so point of inflection

-4	-3.4	-1
>0	0	<0

Concavity changes so point of inflection



y
x
6
4
2
−2
−6
−4
−2
2
$y = x^2e^x$
Local Maximum
$\left(-2, \frac{4}{e^2}\right)$
Point of Inflection
where
$x = -2 - \sqrt{2}$
y Intercept and
Local Minimum
(0 , 0)
Point of Inflection
where
$x = -2 + \sqrt{2}$

Q32

Let (x, x^2+3x+5) be the point of tangent on the curve

$m=\dfrac{x^2+3x+5}{x}$ by gradient formula between two points

$m=2x+3$ by derivative as gradient function

$$\frac{x^2+3x+5}{x}=2x+3 \Rightarrow x^2=5$$

$$\left[\begin{array}{l} x=\sqrt{5} \Rightarrow y=(2\sqrt{5}+3)x \\ x=-\sqrt{5} \Rightarrow y=(-2\sqrt{5}+3)x \end{array}\right.$$

Q33a

x	0	1	2	3	4
$f(x)$	1	1.2214…	2.2255..	6.0496…	24.5325..

$$\therefore \int_0^4 e^{\frac{t^2}{5}}\,dt \approx \frac{4-0}{2(4)}\left(1+24.5325+2\left(1.2214+2.2255+6.0496\right)\right)$$

$$\therefore \int_0^4 e^{\frac{t^2}{5}}\,dt \approx 22.26275$$

$$\therefore P\big|_{t=4}-P\big|_{t=0}=\int_0^4 9e^{\frac{t^2}{5}}-2t\,dt$$

$$\therefore P\big|_{t=4}=\int_0^4 9e^{\frac{t^2}{5}}-2t\,dt+34$$

$$\therefore P\big|_{t=4}=9\int_0^4 e^{\frac{t^2}{5}}\,dt-t^2\Big|_0^4+34$$

$$\therefore P\big|_{t=4}=9\times 22.26275-16+34$$

$$\therefore P\big|_{t=4}\approx 218.36475$$

$$\therefore P=218, \text{ when } t=4$$

Q33b

$$218=4Ae^{-0.2}-100$$

$$318=4Ae^{-0.2}$$

$$A=\frac{318}{4e^{-0.2}}$$

$$A\approx 97.10151927..$$

$$A=97$$

Q33c	$P(t)=97te^{-0.05t}-100$ $P'(t)=97e^{-0.05t}-\frac{97}{20}te^{-0.05t}$ $P'(t)=0$ $\therefore 97e^{-0.05t}\left(1-\frac{t}{20}\right)=0$ $e^{-0.05t}\neq 0$ $\therefore t=20$

t	**19**	**20**	**21**
$P'(t)$	**1.87569…**	**0**	**-1.6979…**
	increasing	**Stationary**	**decreasing**

$\therefore t=20$ is a maximum

$P(20)=97(20)e^{-1}-100$

$P(20)=613.6861159...$

$\therefore$ max population $=613$